SEM-2 (MATHEMATICS GENERAL)
(ANUP RAY)

Ex: Solve $y = px + p - p^2$

Ans: The given equation $y = px + p - p^2$ --- (1) is a clairaut's form.
Differentiating (1) both sides with respect to x,

$$p = p + x\frac{dp}{dx} + \frac{dp}{dx} - 2p\frac{dp}{dx}$$

or $\frac{dp}{dx}(x + 1 - 2p) = 0$

Either $\frac{dp}{dx} = 0$, integrating $p = c$ ---- (2), c being an arbitrary constant.

otherwise $x + 1 - 2p = 0$ or $p = \frac{x+1}{2}$ --- (3)

Eliminating p between (1) and (2) we get the general or complete solution as
$$y = cx + c - c^2$$

Again eliminating p between (1) & (3) we get the singular solution as

$$y = \frac{1}{2}(x+1)x + \frac{x+1}{2} - \frac{(x+1)^2}{4}$$

or $4y = 2x^2 + 2x + 2x + 2 - x^2 - 2x - 1$

or $4y = x^2 + 2x + 1$

or $4y = (x+1)^2$

Ex: Solve $1 - p^2 = \sin^{-1}(px - y)$

Ans. The given equation can be written as

$$px - y = \sin(1 - p^2)$$

or $y = px - \sin(1 - p^2)$ --- (1)

which is a clairaut's equation.

Differentiating (1) both sides w.r.t. x we have

$$p = p + \frac{dp}{dx} \cdot x - (-2p)\cos(1-p^2)\frac{dp}{dx}$$

or $\frac{dp}{dx}\left\{x + 2p\cos(1-p^2)\right\} = 0$

Either $\frac{dp}{dx} = 0$, integrating, $p = c$ --- (2)
where c being an arbitrary constant.

otherwise $x + 2p\cos(1-p^2) = 0$ --- (3)

Eliminating p between (1) & (2) we get the general or complete soln. as

$$y = cx - \sin(1-c^2)$$

Eliminating p between (1) & (3) we get the singular solution

Ex: Solve the following problems (Home work)

1) $p = \log(px - y)$ 2) $y = px + \frac{a}{p}$

3) $y = px + ap(1-p)$ 4) $py = p^2(x-b) + a$

5) $y = px + p^n$ 6) $y = px + \sqrt{a^2p^2 + b^2}$

7) $(x-a)p^2 + (x-y)p - y = 0$

8) $(y+1)p - xp^2 + 2 = 0$

9) $\cos px \cos y + \sin px \sin y = p$